VERTEX ALGEBRAS AND ASSOCIATED SCHEMES

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ANNE MOREAU

The goal of this series of lectures is to introduce the theory of vertex algebras, with emphasis on their geometrical aspects.

Roughly speaking, a vertex algebra is a vector space V, endowed with a distinguished vector, the vacuum vector, and the vertex operator map from V to the space of formal Laurent series with linear operators on V as coefficients. These data satisfy a number of axioms. Although the definition is purely algebraic, these axioms have deep geometric meaning. They reflect the fact that vertex algebras give an algebraic framework of the two-dimensional conformal field theory.

To each vertex algebra V one can naturally attach a certain Poisson variety X_V called the associated variety of V. A vertex algebra V is called lisse if $\dim X_V = 0$. Lisse vertex algebras are natural generalizations of finite-dimensional algebras and possess remarkable properties. For instance, the characters of simple V-modules form vector valued modular functions. More generally, vertex algebras whose associated variety has only finitely symplectic leaves, are also of great interest for several reasons that will be addressed in the lectures.

It is only quite recently that the study of associated varieties of vertex algebras and their arc spaces, has been more intensively developed. In this mini-course I wish to highlight this aspect of the theory of vertex algebras which seems to be very promising. In particular, I will include open problems on associated varieties in the setting of affine vertex algebras (vertex algebras associated with Kac-Moody algebras) and W-algebras (they are certain vertex algebras attached with nilpotent elements of a simple Lie algebra) raised by my recent works with Tomoyuki Arakawa.

References

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