Characters of finite groups of Lie type: open problems and conjectures

Meinolf Geck, Dijon, 3-7 July 2017

The starting point of these lectures are certain long-standing conjectures in the character theory of finite groups. Given the classification of all finite simple groups, one can try to attack these problems through reduction theorems and an investigation of the finite simple groups. In recent years, this program has led to substantial advances; see, for example, Malle's survey [4]. Typically, the simple groups of Lie type are the hardest case to deal with in this set-up. These groups arise as fixed point sets of (connected, reductive) algebraic groups under Frobenius maps and, following the work of Deligne and Lusztig [1], [3], the character theory of these groups can be approached by geometric methods. We introduce these geometric methods and survey the current state of knowledge.

Lecture 1: Problems on characters of finite groups
Lecture 2: The virtual characters of Deligne and Lusztig
Lecture 3: Lusztig's classification of irreducible characters
Lecture 4: Examples, outlook, open questions

References

[1] P. Deligne and G. Lusztig, Representations of reductive groups over finite fields, Annals Math. **103** (1976), 103–161.

[2] M. Geck, A first guide to the character theory of finite groups of Lie type, `arXiv:1705.05083`.

[3] G. Lusztig, Algebraic and geometric methods in representation theory (Lecture at the Chinese University of Hong Kong, Sep. 25, 2014), `arXiv:1409.8003`.

[4] G. Malle, Local-global conjectures in the representation theory of finite groups, *in: Representation theory – current trends and perspectives*, EMS Ser. Congr. Rep., pp. 519–539, Eur. Math. Soc., Zürich, 2017.